WAVEGUIDES OF ARBITRARY CROSS SECTION BY SOLUTION OF A NONLINEAR INTEGRAL EIGENVALUE EQUATION

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Abstract

An accurate solution to the problem of wave propagation in conducting waveguides of arbitrary cross section is developed. The solution is available in the form of two digital computer programs, which yield computed values of both cutoff wave numbers and field distributions. Sample calculations are presented for the ridge waveguide.

Summary

I. Formulation

The problem of electromagnetic wave propagation in hollow, conducting waveguides of arbitrary cross section is considered. The problem is formulated as the integro-differential equation

$$L(\vec{J}) = 0 \tag{1}$$

where L is the integro-differential operator

$$L(\vec{J}) = -\vec{E}_{tan} = [j\omega\vec{A} + \vec{\nabla}\varphi]_{tan} \tag{2}$$

Here, $\vec{A}$, φ, and $\vec{E}$ are the vector potential, scalar potential, and electric field, respectively, due to the two-dimensional wall current $\vec{J}$. tan denotes the component of electric field tangential to the waveguide wall.

Equation (1) has been shown to be an expression of a nonlinear eigenvalue problem.[1] The waveguide cutoff wave numbers $\{k_n\} = \{k_1, k_2, \ldots\}$ and corresponding wall currents $\{\vec{J}_n\} = \{\vec{J}_1, \vec{J}_2, \ldots\}$ appear as eigenvalues and eigenfunctions, respectively.

II. Reduction to Matrix Formulation

A variational solution is effected by reducing equation (1) to matrix form using the method of moments.[2] The problem is thereby reduced to the matrix equation

$$[Z]\,[I] = [0] \tag{3}$$

where the elements of the generalized impedance matrix [Z] are determined purely by the waveguide geometry and the expansion and testing functions employed in the moment solution. The solution is then simplified by symmetry considerations. The details of this development are presented elsewhere.[1]

III. Determination of Cutoff Wave Numbers and Field Distributions

If a complete expansion function set on the domain of the operator L is used in the development of equation (3), then solutions exist if and only if

$$|\text{Det } Z(k)| = 0 \tag{4}$$

In general, the expansion set used here will enable only an approximation of exact wall currents. For an adequate approximation, the cutoff condition is characterized by

$$|\text{Det } Z(k)| = \text{Minimum} \tag{5}$$

It has been shown that once the cutoff wave numbers are determined the corresponding wall current can be computed approximately as the eigenvector of a symmetric real matrix.[1] The method used here is to first perform a Householder tridiagonalization. The eigenvalues are located by bisection, using Sturm sequences, and the eigenvectors are computed using Wielandt iteration.

The expressions used in the field evaluation scheme used here were derived in detail in reference 1. Any field component at a point P on a waveguide cross section can be expressed in the form

$$\text{Field } (P) = \oint_C \vec{E}^r \cdot \vec{J}\, d\ell \tag{6}$$

where $\vec{E}^r$ is a known function, C is the contour bounding the waveguide cross section. The field distributions are computed by numerically integrating equation (6) with the wall currents being approximated as previously described.

The accuracy of this method has been demonstrated using computations made for rectangular and circular waveguides, where exact solutions are known.[1] Using relatively low order matrices cutoff wave numbers are determined to within a few tenths of one per cent of exact values, while computed distributions of both longitudinal and transverse field components agree to within a few per cent of exact values.

IV. Example--Ridge Waveguide

Exact solutions for the single ridge waveguide shown in FIG.1 are not known. For this reason values of cutoff wave numbers computed by the method developed here will be compared to other estimates of the exact cutoff wave number. In particular, results here are shown for the lowest four TE modes and the lowest TM mode. Values tabulated in Table I correspond to the waveguide in FIG.1 with dimensions in the proportion a:b:c:d = 2:4:2:1. It is to be noted that the waveguide in FIG.1 is symmetric about the X-axis. Accordingly, the modes tabulated have wall currents possessing odd or even symmetry about this axis and are so labeled ODD or EVEN in Table I. In this table the values computed by the method developed here are compared to various reference values. Tabulated immediately to the right of each reference value is the per cent difference between that reference value and the corresponding computed value. Computed values of field distributions in the ridge waveguide are presented in reference 1.

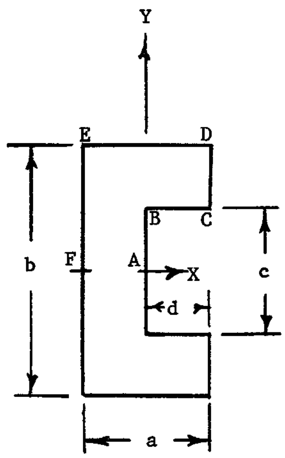

FIG.1

SINGLE RIDGE WAVEGUIDE CROSS SECTION

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Table I--Normalized Cutoff Wave Numbers (kb) for the Single Ridge Waveguide

MATRIX ORDER	MODE	WAVE NUMBER (kb)		PERCENT DIFFERENCE	REFERENCE[3] WAVE NO.	PERCENT DIFFERENCE	REFERENCE[4] WAVE NO.	PERCENT DIFFERENCE
		COMPUTED	REFERENCE					
13	TE^{ODD}	2.2566	2.250*	0.31	2.2627	0.27	2.2412	0.68
15	TE^{EVEN}	4.9373	4.8404*	2.0	4.9251	0.25	4.8460	1.9
13	TE^{ODD}	6.5218	6.4575**	0.99	6.4864	0.54	6.4532	1.0
22	TE^{EVEN}	7.5361	7.5074**	0.38	7.5249	0.15	7.5188	0.23
29	TE^{EVEN}	12.164	11.974**	1.6	-	-	12.1416	0.18

*Value given in references[5],[6]

**Value given in reference[7]